Exercise Session for Financial Data Analysis
Summer term 2011
Problem Set 2

Write to `haas@stat.uni-muenchen.de` if you want to present. You can also indicate multiple exercises (ordered according to your preference) in case your most preferred problem has already been assigned.

Problem 1 We consider weekly observations of the MSCI World stock market index from January 1990 to May 2009 ($T = 1012$ observations).[1] Returns are calculated as

$$r_t = 100 \times \log(I_t/I_{t-1}), \tag{1}$$

where I_t is the index level at time t. Returns are calculated from Wednesday to Wednesday.

Several properties of the index and the index returns are displayed in Table 1 and Figures 1 and 2.

(a) Describe what you observe in Table 1 and Figures 1 and 2 insofar as it is typical for many financial return series.

Table 1: Basic statistics of MSCI returns.

Mean	Variance	Skewness	Kurtosis
0.051	4.81	–0.79	8.06

"Skewness" denotes the moment–based coefficient of skewness, $\gamma = m_3/m_2^{3/2}$, and "Kurtosis" the moment–based coefficient of kurtosis, $\kappa = m_4/m_2^2$, where $m_i = T^{-1}\sum_t (r_t - \bar{r})^i$, $i = 2, 3, 4$, and $\bar{r} = T^{-1}\sum_t r_t$.

Problem 2 Explain the Jarque–Bera test for normality. For the return data considered in Problem 1, perform the test at a significance level of $\alpha = 0.05$. (Table 2 may be helpful in this regard.)

[1]For more information on the index, see, e.g., `http://en.wikipedia.org/wiki/MSCI_World`.

Problem 3 Consider a stationary Gaussian ARMA(7,3) process,

$$Y_t = \phi_0 + \sum_{i=1}^{7} \phi_i Y_{t-i} + \epsilon_t + \sum_{i=1}^{3} \theta_i \epsilon_{t-i}, \quad \epsilon_t \overset{iid}{\sim} \text{Normal}(0, \sigma^2).$$

Find the kurtosis of the unconditional distribution of Y_t. (Hint: The kurtosis of the iid Gaussian innovation ϵ_t is 3).

Problem 4 (kurtosis of GARCH(1,1) with nonnormal innovations) Consider the GARCH(1,1) process,

$$\epsilon_t = \sigma_t \eta_t \tag{2}$$

$$\sigma_t^2 = \omega + \alpha_1 \epsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2, \tag{3}$$

where η_t is distributed iid with zero mean, unit variance (i.e., $\mathrm{E}(\eta_t^2) = 1$), and kurtosis (fourth moment) $\mathrm{E}(\eta_t^4) = \kappa_4$.

(i) Find the condition for the existence of a finite fourth moment of the process.

(ii) Calculate the unconditional kurtosis of the GARCH process (2)–(3).

Problem 5 Consider the following model for the returns, r_t, of a stock market index:

$$r_t = \mu + \delta \sigma_t + \epsilon_t \tag{4}$$

$$\epsilon_t = \eta_t \sigma_t, \quad \eta_t \overset{iid}{\sim} \mathrm{N}(0, 1) \tag{5}$$

$$\sigma_t^2 = \omega + \alpha_1 \epsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2. \tag{6}$$

Is this an economically plausible specification for the conditional mean return, as specified by (4)? What is the expected sign of parameter δ? What would you expect the constant μ in (4) to be (consider what happens if $\sigma_t^2 = 0$)?

Problem 6 Consider the GARCH(1,1) process

$$\begin{aligned}\epsilon_t &= \eta_t \sigma_t, \quad \eta_t \overset{iid}{\sim} \mathrm{N}(0,1) \\ \sigma_t^2 &= \omega + \alpha_1 \epsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2.\end{aligned}$$

We want to calculate multi–step ahead variances. That is, for $\tau \geq 1$, find

$$\mathrm{Var}(\epsilon_{t+\tau}|I_t) = \mathrm{E}(\epsilon_{t+\tau}^2|I_t), \tag{7}$$

where $I_t = \{\epsilon_s : s \leq t\}$ is the information set at time t. If we have a model for daily returns and we are interested in, for example, weekly or monthly returns, we also need quantities of the form

$$\mathrm{Var}(\epsilon_{t+1} + \cdots + \epsilon_{t+\tau}|I_t). \tag{8}$$

Problem 7 Suppose that the return r_t of your portfolio is generated by

$$\begin{aligned}r_t &= 0.025 + \epsilon_t \\ \epsilon_t &= \eta_t \sigma_t, \quad \eta_t \overset{iid}{\sim} \mathrm{N}(0,1) \\ \sigma_t^2 &= 0.025 + 0.075\epsilon_{t-1}^2 + 0.9\sigma_{t-1}^2.\end{aligned}$$

Your current estimate for σ_t^2 is its unconditional expectation. Unfortunately, however, due to unpredictable adverse market conditions, your portfolio suffers from an unusually large negative shock, so that $r_t = -4.75$. Calculate the 1% Value–at–Risk for period $t + 1$.

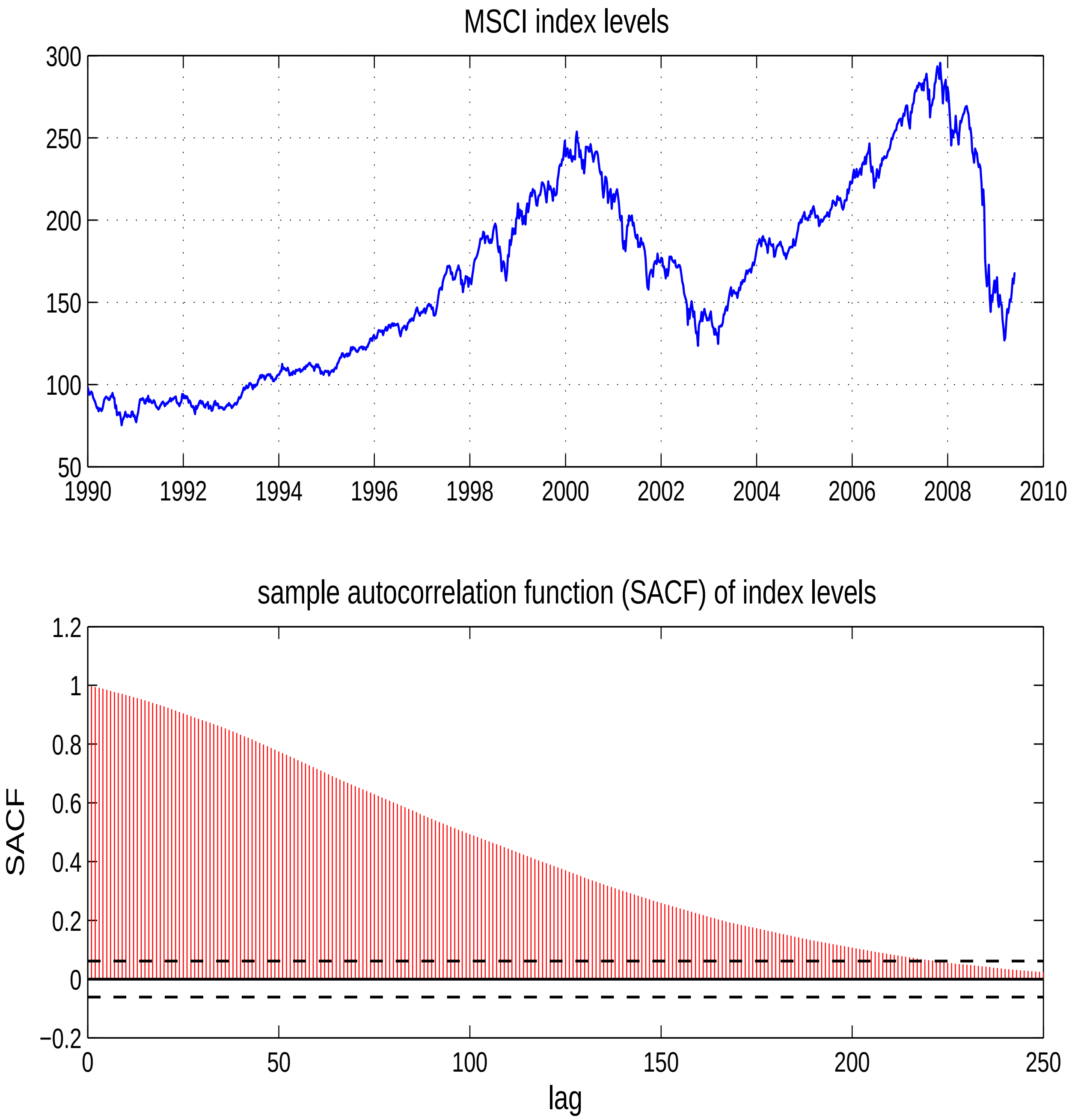

Figure 1: MSCI index levels.

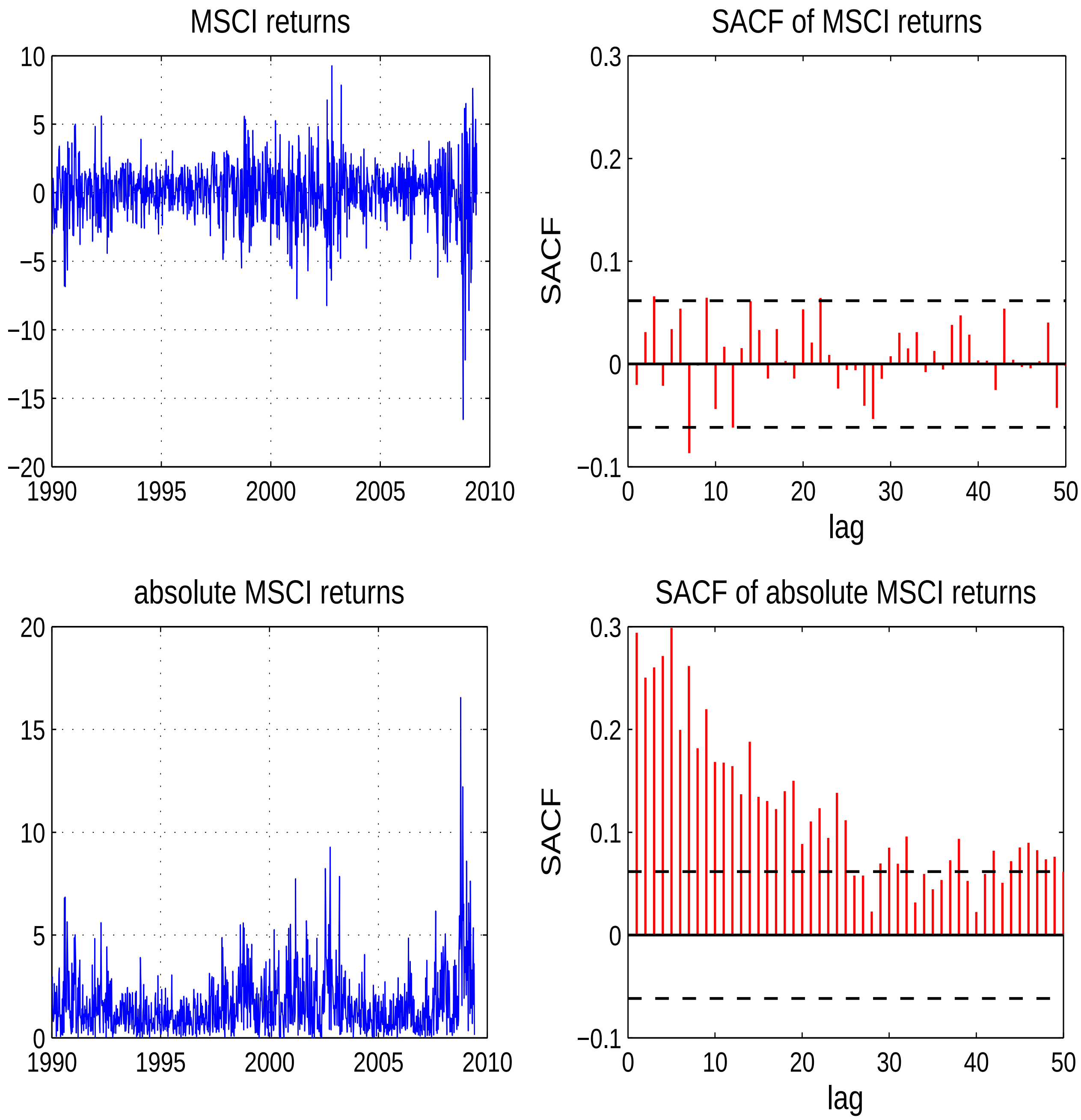

Figure 2: MSCI index returns.

Table 2: Quantiles of the χ^2 distribution.

ν	$z_{0.9}$	$z_{0.95}$	$z_{0.975}$	$z_{0.99}$
1	2.7055	3.8415	5.0239	6.6349
2	4.6052	5.9915	7.3778	9.2103
3	6.2514	7.8147	9.3484	11.3449
4	7.7794	9.4877	11.1433	13.2767
5	9.2364	11.0705	12.8325	15.0863
6	10.6446	12.5916	14.4494	16.8119
7	12.0170	14.0671	16.0128	18.4753
8	13.3616	15.5073	17.5345	20.0902
9	14.6837	16.9190	19.0228	21.6660
10	15.9872	18.3070	20.4832	23.2093

ν denotes the degrees of freedom of the χ^2 distribution, and z_α is the α–Quantile, that is, z_α is such that

$$\int_0^{z_\alpha} \chi^2(z;\nu)dz = \alpha, \tag{9}$$

where $\chi^2(z;\nu)$ is the density function of a χ^2 random variable with ν degrees of freedom.